Inverse Functions and Logarithms

Note Title 14/09/2008

Section 1.6 Stewart

ASSIGNMENT 1 DUE TUES 8:30 AM (23 hours :-)

All review problems from Appendix D

Hand in #24, 30, 46, and 50.

Inverse functions

Recall 4 ways to think of functions: tabular, formulaic, graphical, and algorithmic (text has "verbal", pfui on verbal descriptions!).

Tabular function

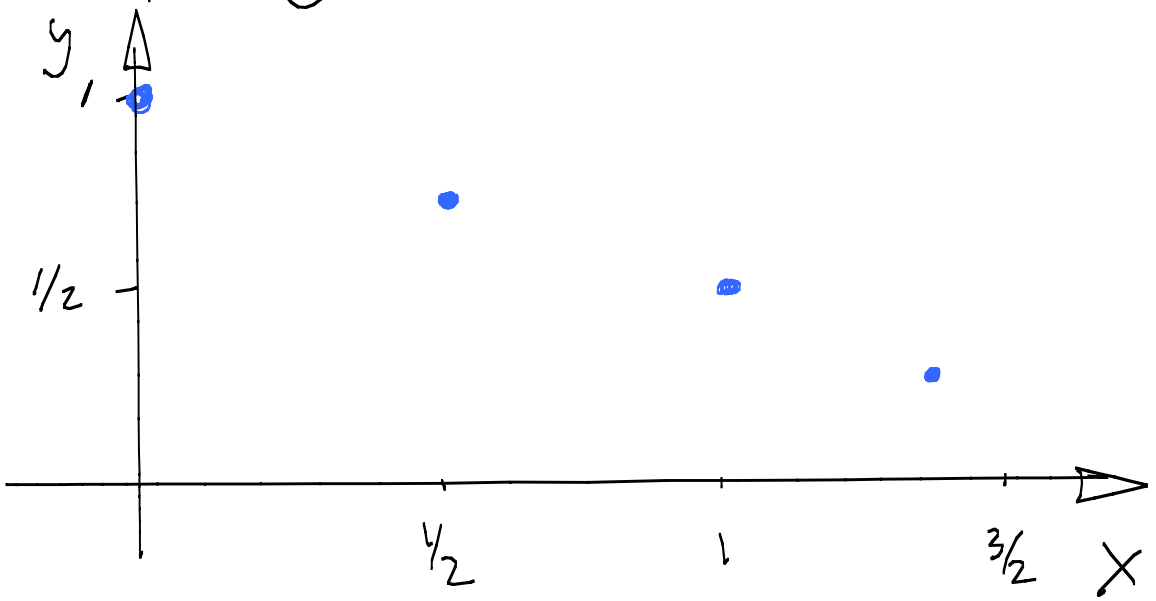
x	y
0	1
$1/2$	$1/\sqrt{2}$
1	$1/2$
$\sqrt{2}$	$1/4$

its inverse

y	x
0	1
$1/2$	$1/\sqrt{2}$
1	$1/2$
$\sqrt{2}$	$1/4$

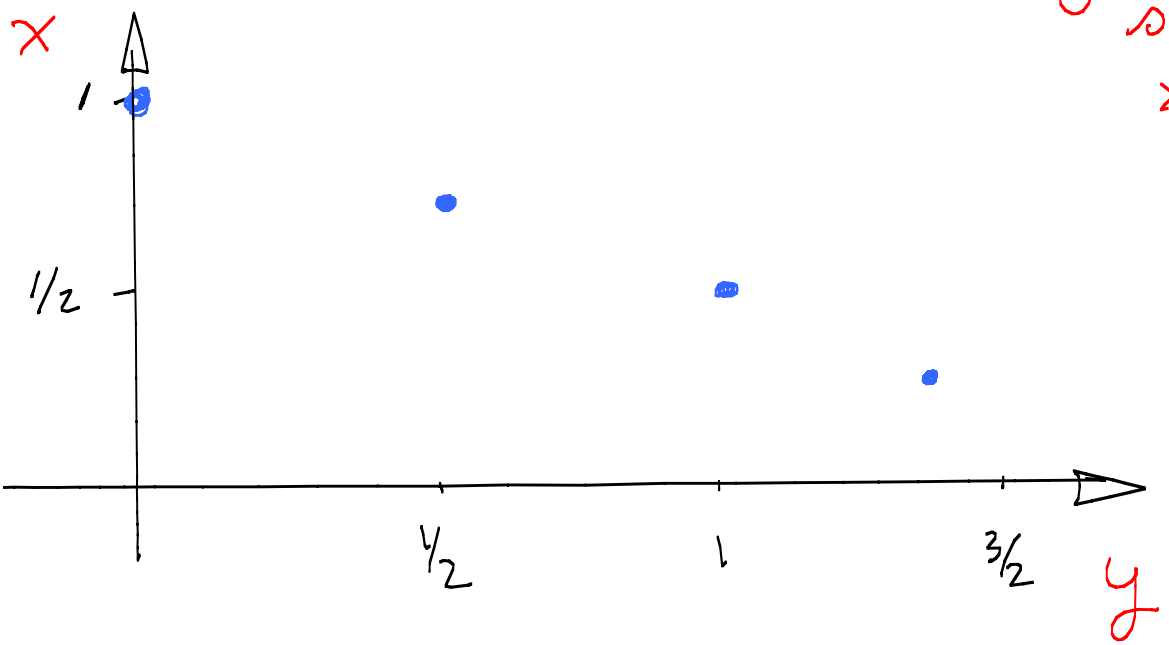
and that's it, just interchange y and x.

Graphing this function:

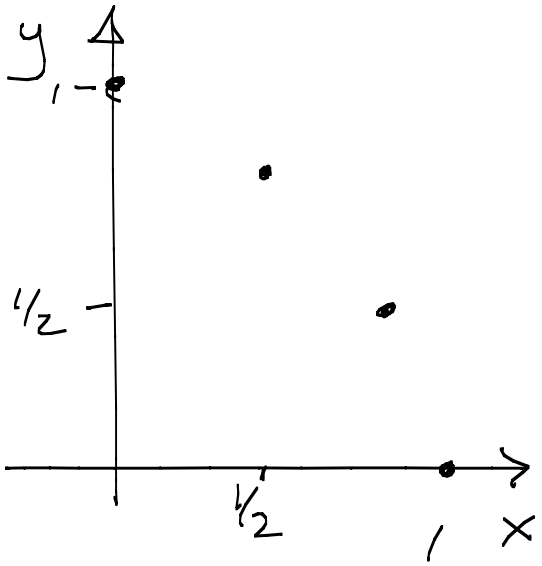


Inverse Function

just swap x & y



By convention we plot the independent variable horizontally (but it's just a convention) If we flip that last graph about $y = x$ we get the more familiar



But the key thing is to interchange x and y.

Viewed algorithmically

$y =$

if $x = 0$ then
    return 1
elif $x = 1/2$ then
    return $1/\sqrt{2}$
elif $x = 1$ then
    return $1/2$
elif $x = \sqrt{2}$ then
    return $1/4$
else
    return Ω
end if

$y =$

if $x = 1$ then
    return 0
elif $x = 1/\sqrt{2}$ then
    return $1/2$
elif $x = 1/2$ then
    return 1
elif $x = 1/4$ then
    return $\sqrt{2}$
else
    return Ω
end if

Note that an algorithm must be definite: $x = 0 \Rightarrow y = 1$, not "make a choice" of multiple values.

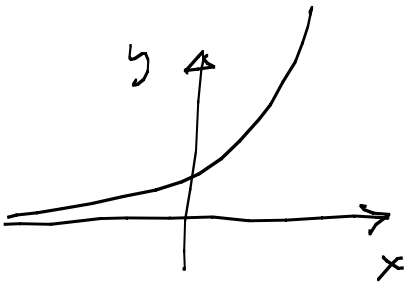
Formulas

$y = e^x$ inverse $x = e^y$

which we can write

$y = \ln x$.

(by definition of $\ln$)



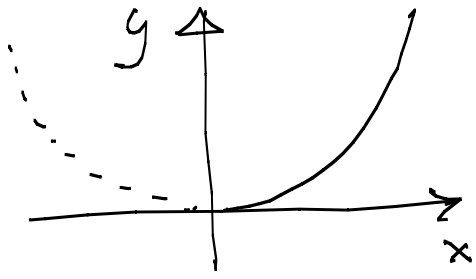
$y = x^2$ inverse $x = y^2$

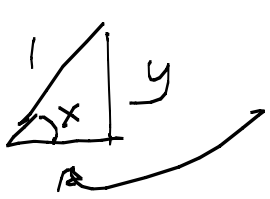
not 1-1

need to choose a branch

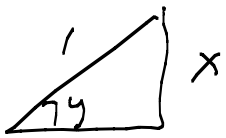
$+\sqrt{x} = y$

principal branch





$\therefore = \sqrt{1-y^2}$



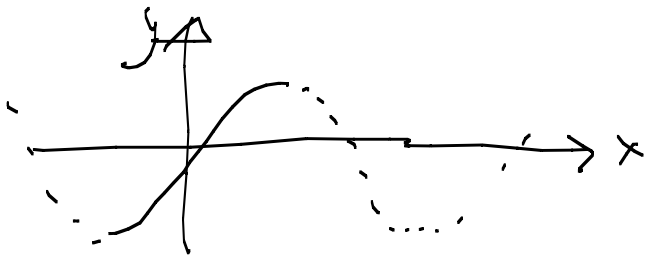
$y = \sin x$ inverse $x = \sin y$

we write this as

$\sin^{-1} x = y$

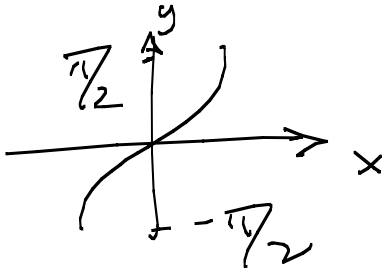
(invsin $x = y$ would have been better.)

also seen arcsin asin



Branch choice:

$-\pi/2 \leq \sin^{-1} x \leq \pi/2$



$y = \cos x \Rightarrow$ inverse is $x = \cos y$

$y = \tan x$ " " $x = \tan y$

"Verbally"

"independent" variable is the one we can choose

eg $x = 0$

(then y must be 1)

"The value of y depends on the value of x"

[Math to English "faux amc",

almost: Consider the function at left. Mathematically, y depends on x, but not in ordinary usage of English.]

x	y
0	17
$\frac{1}{2}$	17
1	17
$\sqrt{2}$	17

In going to the inverse function, we are 'merely' switching points of view, and allowing choice of y to force the corresponding choice of x.

Eg $y = 2^x \Rightarrow$ inverse is $x = 2^y$

or $y = \lg x$

Don't make this harder than it is - it's just "swap x and y".

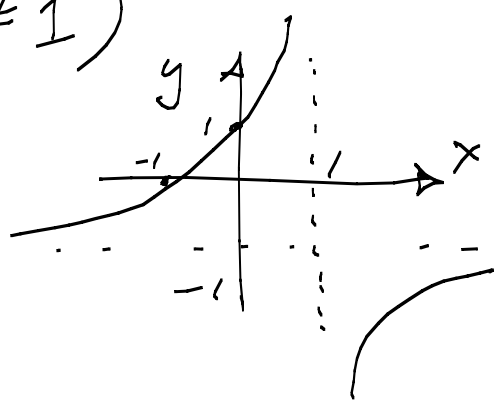
$(\lg = \log_2)$

$x = 8$?

$\Rightarrow y = 3$.

Sometimes one can simplify

$$y = \frac{1+x}{1-x} \qquad (x \neq 1)$$



Inverse Function

$$x = \frac{1+y}{1-y} \qquad \text{now } y \neq 1$$

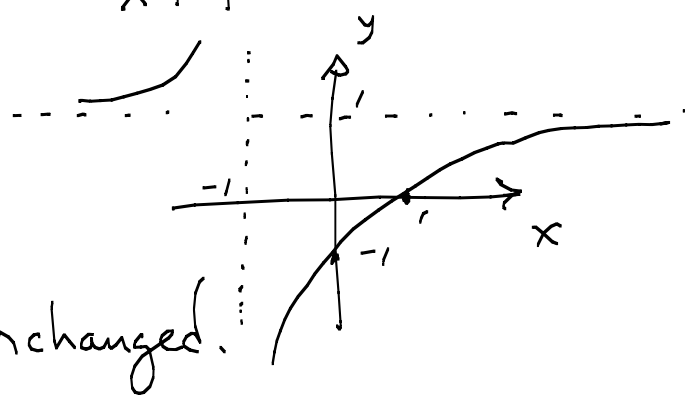
we can simplify this

$$x(1-y) = 1+y$$

$$x - xy = 1+y$$

$$x - 1 = xy + y = (x+1)y$$

$$\therefore \quad y = \frac{x-1}{x+1}$$



Same graph,
x and y interchanged.